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Thermodynamic properties of spontaneous magnetization in Chern-Simons QED_3

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Abstract

The spontaneous magnetization in Chern-Simons QED_3 is discussed in a finite temperature system. The thermodynamical potential is analyzed within the weak field approximation and in the fermion massless limit. We find that there is a linear term with respect to the magnetic field with a negative coefficient at any finite temperature. This implies that the spontaneous magnetic field does not vanish even at high temperature. In addition, we examine the photon spectrum in the system. We find that the bare Chern-Simons coefficient is cancelled by the radiative effects. The photons then become topologically massless according to the magnetization, though they are massive by finite temperature effects. Thus the magnetic field is a long-range force without the screening even at high temperature.

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1 Introduction

The 2+1 dimensional gauge field theories with Chern-Simons (C-S) term have had a lot of motivation to study. In addition to their own theoretical interests, they are expected to describe the physics of the planar systems like the quantum Hall effect [1, 2] and the high T_C superconductivity [3]. Many interesting and remarkable properties of these theories have been found in past a decade, such as their chiral dynamics [4, 5], the spontaneous parity violation by radiatively induced C-S terms [6]. A few years ago, it was also found that in a model of three dimensional quantum electrodynamics (QED_3) with a bare C-S term the spontaneous magnetic field can be stable by the fermion loop effects, which eventually triggers the dynamical Lorentz symmetry breaking (Hosotani [7]). He has calculated the energy densities as a function of the magnetic field B in the fermion one-loop level and showed that the minimum of the energy density can be at $B \neq 0$ under a consistency condition for non-zero magnetic fields. It has been found that this condition is also regarded as the sufficient condition for the spontaneous generation of the magnetic field as well [8]. According to the spontaneous Lorentz symmetry breaking, the physical photon, which is topologically massive in the tree level due to the bare C-S term, becomes massless as a Nambu-Goldstone boson. This mechanism of the spontaneous magnetization has been extended into the cases with massive fermions with non-zero fermion density [9]. Even in the case of no bare C-S term, the spontaneous magnetization can occur by adding heavy fermions because the C-S term induced by them can play the same role as the bare C-S term [10]. The spontaneous magnetic field is expected to be important in the connection with the problem of the fractional quantum Hall effect [2].

In this paper, the thermodynamic properties of the spontaneous magnetization in QED_3 with a bare C-S term are investigated. Some of the thermodynamic properties of the model without external fields have been studied to some extent in the different contexts so far. The specific heat for different values of the statistical angle is analyzed in ref. [11]. The restoration of the parity at high temperature limit has been discussed in ref. [12], in which it has been found that the radiatively induced parity violating C-S term vanishes only in the high temperature limit. Our aim here is to examine whether the spontaneous magnetic field, which is dynamically generated at $T = 0$, vanishes or not in a high temperature region. The model we are

studying here is based on ref. [7], in which all fermions are very light so that a non-zero bare C-S term is needed in the Lagrangian for the magnetization [10]. In the finite temperature system, the energy density is replaced by the thermodynamical potential. The quantum fluctuation of the thermodynamical potential is investigated in the fermion one-loop level here. The coefficient of the linear term with respect to the magnetic field in the potential is calculated as a function of temperature [3] as well as the classical part of the potential. Negative values of the coefficient mean the existence of the spontaneous magnetic field at finite temperature. We find that the coefficient becomes more (negative) large behaving like $\sim -\log T$ in high temperature region. Thus the spontaneous magnetic field does not vanish even at high temperature.

In addition, the spectrum of photons is also investigated in the finite temperature system. At zero temperature, the photons are topologically massive at tree level. The mass amounts to the C-S coefficient. If the magnetization occurs, the bare coefficient of the C-S term is exactly cancelled by the radiative effects [14, 15], so that the photons becomes massless. Therefore the photons are regarded as the Nambu-Goldstone bosons of the dynamical breakdown of the Lorentz symmetry [7, 8]. On the other hand, at finite temperature, the Lorentz symmetry is explicitly broken by thermal effects. The photons are then no longer massless even if the magnetization occurs. In general, they eventually have the thermal masses [16, 17, 18]. The magnetic part of the field strength, however, become statically massless and the magnetic force is a long-range force without the screening [12] if the topological mass is cancelled out. We find that, in our model with massless fermions, the bare topological mass is completely cancelled by the radiative effects even at finite temperature if the magnetization occurs. This implies that the static mass of magnetic fields vanishes according to the magnetization even at high temperature. This is consistent with the result of the non-vanishing magnetic field by the thermodynamic potential approach. It is also found that in the case of massive fermions the topological mass is cancelled out only in the 'chirally symmetric' cases with massive fermions.

Furthermore we naively realize the reason of the spontaneous magnetiza-

[3]In our previous preprint [13], some insufficient treatment has been done especially in the evaluation of the contribution of the zero-mode to the coefficient. These are correctively changed in the first half of this paper.

tion from the viewpoint of the difference of the free energies of free photons between $B \neq 0$ and $B = 0$, where B is the magnetic field. It is found from rough estimation that the linear $|B|$ term with a negative coefficient appears in the difference. This explains the magnetization at finite temperature to some extent. These characteristics of the magnetization at finite temperature might be interesting in the connection with the physics such as the high T_C superconductivity.

This paper is organized as follows. In the next section, some general properties and statistic characteristics of the model will be presented. In Sec 3, the spontaneous magnetization in this model in $T = 0$ case will be reviewed briefly for a preparation for $T \neq 0$ cases in order to clarify the essential point for the magnetization and also to fix some notations. The strategy to extend the investigation to the finite temperature system will be explained then. Sec. 4 will be devoted to the calculation of self-energies of gauge bosons in the one-loop level and the weak-field approximation. In Sec. 5, the results that the spontaneous magnetic field does not vanish will be presented in the fermion massless limit by analyzing the effective potential both analytically and numerically. In Sec. 6, the spectrum of the physical photons will be examined in the finite temperature system. In Sec. 7, the reason why the spontaneous magnetic field does not vanish will be discussed. Also some comments on our results will be in order. In the last section, we will summarize our results. Some definitions, details of derivations and formulas are summarized in Appendices.

2 Model in Finite Temperature System

We consider the model of QED_3 with a bare Chern-Simons term described by the Lagrangian

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{\kappa}{2}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho + \sum_a \bar{\psi}_a \left\{\gamma_a^\mu(i\partial_\mu + q_a A_\mu) - m_a\right\}\psi_a\,, \tag{1}$$

where gamma matrices are defined as $\gamma_a^\mu \equiv (\eta_a\sigma^3, i\sigma^1, i\sigma^2)$. There can be two types of photons according to the choice of $\eta_a \equiv (i/2)\mathrm{tr}\gamma_a^0\gamma_a^1\gamma_a^2 = \pm 1$. Since the model (1) has charge conjugation invariance, we can take the electric charges q_a to be positive without loss of generality. Also since the transformation $m_a \leftrightarrow -m_a$ is equivalent to $\eta_a \leftrightarrow -\eta_a$, we can consider m_a as

non-negative. We then call the fermion with $\eta_a = \pm 1$ as $\eta_\pm$-fermion, respectively.

Under the existence of a classical magnetic field B ($\langle 0|A^\mu|0\rangle = -\delta^{\mu 1} x^2 B$), the energy spectrum for the η_+ fermion takes the form [19]

$$\begin{array}{l} E_0^a = \epsilon(q_a B) \times \omega_0^a, \\ E_n^a = \pm\, \omega_n^a, \quad (n \geq 1), \end{array} \tag{2}$$

where $\epsilon(x)$ is the usual sign function, $\omega_n^a = \sqrt{m_a^2 + 2n/l_a^2}$ and $l_a^2 = 1/|q_a B|$. Then the general solution of the Dirac equation is

$$\psi_a(x) = \sum_{n,p} a_{np} \left\{ \begin{array}{c} u_{np}^a(x) \\ w_{np}^{ac}(x) \end{array} \right\} + \sum_{n,p} b_{np}^\dagger \left\{ \begin{array}{c} w_{np}^a(x) \\ u_{np}^{ac}(x) \end{array} \right\}, \quad \begin{array}{c} (q_a B > 0) \\ (q_a B < 0) \end{array}, \tag{3}$$

where $u_{np}^c = U_c \bar{u}_{np}^t, U_c = \gamma_2$ and the same for w_{np}^c. The positive energy solution u_{np} and the negative energy solution w_{np} in eq. (3) are

$$u_{0p}(x) = \frac{1}{\sqrt{lL_1}} e^{-i(\omega_0 t + kx^1)} \begin{pmatrix} v_0(\xi) \\ 0 \end{pmatrix}, \tag{4}$$

$$u_{np}(x) = \frac{1}{\sqrt{lL_1}} e^{-i(\omega_n t + kx^1)} \frac{1}{\sqrt{2\omega_n}} \begin{pmatrix} \sqrt{\omega_n + m}\, v_n(\xi) \\ -i\sqrt{\omega_n - m}\, v_{n-1}(\xi) \end{pmatrix}, (n \geq 1), \tag{5}$$

$$w_{np}(x) = \frac{1}{\sqrt{lL_1}} e^{+i(\omega_n t - kx^1)} \frac{1}{\sqrt{2\omega_n}} \begin{pmatrix} \sqrt{\omega_n - m}\, v_n(\xi) \\ i\sqrt{\omega_n + m}\, v_{n-1}(\xi) \end{pmatrix}, \ (n \geq 1), \tag{6}$$

where we dropped the fermion index a for simplicity, and $k = 2\pi p/L_1$ (p: integer), $\xi = x^2/l - lk$ and $v_n(\xi) = (-1)^n \pi^{-1/4} (2^n n!)^{-1/2} e^{\xi^2/2} d^n/d\xi^n e^{-\xi^2}$. The solution for η_- fermion is obtained by making the transformation ($t \to -t$) from corresponding η_+ cases. There is an asymmetry between positive and negative energy solutions in the lowest Landau levels. The vacuum expectation value of the total charge is (see eq. (A.2))

$$\langle 0|\hat{Q}|0\rangle = \sum_a q_a N_s^a \eta_a \epsilon(B) \left(\nu_a - \frac{1}{2}\right), \tag{7}$$

where the number of degeneracy is given by $N_s^a = \sum_p 1 = |q_a B|/2\pi \times \int d^2x$ and we introduced the filling factor $\nu_a \equiv \langle 0|\hat{\nu}_a|0\rangle$ for each fermion ψ_a,

$$\hat{\nu}_a = \left\{ \begin{array}{ll} \sum_p a_{0p}^\dagger a_{0p}/N_s^a, & (\eta_a \epsilon(B) > 0), \\ \sum_p b_{0p}^\dagger b_{0p}/N_s^a, & (\eta_a \epsilon(B) < 0). \end{array} \right. \tag{8}$$

We here consider the completely filled ($\nu_a = 1$) or the empty ($\nu_a = 0$) cases for zero temperature. From the time component of the equation of motion with the charge, we have a relation $\int d^2x\kappa B = \langle 0|\hat{Q}|0\rangle$. The consistency condition for $B \neq 0$ is then obtained from this relation and eq. (7) as

$$\kappa = \sum_a \frac{\eta_a q_a^2}{2\pi}\left(\nu_a - \frac{1}{2}\right). \tag{9}$$

In this paper, we would like to study this model in the finite temperature system. The partition function for the fermion sector is given by

$$\begin{aligned} Z &= \mathrm{tr} e^{-\beta(\hat{H}-\mu\hat{Q})} \\ &= \prod_a \left(e^{-\beta\omega_0^a} e^{\frac{1}{2}\beta\mu_a q_a \eta_a \epsilon(B)} + e^{-\frac{1}{2}\beta\mu_a q_a \eta_a \epsilon(B)}\right)^{N_s^a} \\ &\quad \times \prod_{n=1}^{\infty}\left(1 + e^{-\beta(\omega_n^a - \mu_a q_a)}\right)^{N_s^a}\left(1 + e^{-\beta(\omega_n^a + \mu_a q_a)}\right)^{N_s^a} e^{\beta\omega_n^a N_s^a}, \end{aligned} \tag{10}$$

where $\beta = 1/T$ and $\mu\hat{Q} = \sum_a \mu_a \hat{Q}_a$ (see eqs. (A.1) and (A.2)). The total charge $Q = \sum_a Q_a$ in the finite temperature system is obtained by using (10),

$$Q_a = \frac{1}{\beta}\frac{\partial \ln Z}{\partial \mu_a}. \tag{11}$$

Note that our definition of the chemical potential is a little different from the ordinary one and $q_a\mu_a$ in our notation is normally called the chemical potential in the literature. The condition for $B \neq 0$ in the finite temperature system is given from the time component of the equation of motion as $\kappa = Q/(VB)$, where V is the two-dimensional volume. In the massless limit ($m_a \to 0$), this becomes by using eqs. (10) and (11)

$$\begin{aligned} \kappa &= \epsilon(B)\sum_a \frac{q_a^2}{2\pi}\left\{\frac{1}{2}\tanh\left(\frac{1}{2}q_a\beta\mu_a\right)\right. \\ &\quad \left. + \sum_{n=1}^{\infty}\left(\frac{1}{e^{\beta(\omega_n^a - q_a\mu_a)} + 1} - \frac{1}{e^{\beta(\omega_n^a + q_a\mu_a)} + 1}\right)\right\}. \end{aligned} \tag{12}$$

The chemical potential μ_a is determined as a function of T and B by eq. (12) or by the following relations which are induced from eqs. (9) and (12):

$$\begin{aligned} 1 &= \tanh\left(\frac{1}{2}q_a\beta|\mu_a|\right) \\ &\quad + 2\sum_{n=1}^{\infty}\left(\frac{1}{e^{\beta(\omega_n^a - q_a|\mu_a|)} + 1} - \frac{1}{e^{\beta(\omega_n^a + q_a|\mu_a|)} + 1}\right), \end{aligned} \tag{13}$$

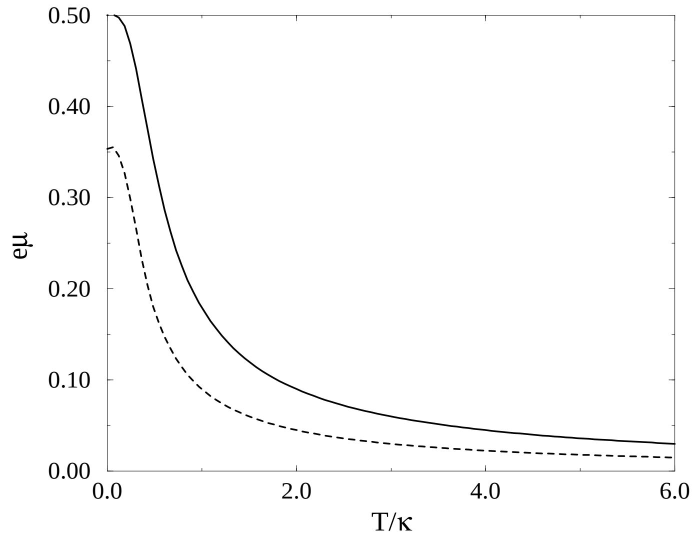

Figure 1: The chemical potential $|\mu_a|$ in the fermion massless limit. By assuming the consistency condition (12), μ become a function of B and T. It can be seen that $|\mu_a|$ is a monotonous decreasing function of T with the definite magnetic field $|B|$. The solid line is the case of $|q_a B|/\kappa^2 = 1/2$. The dashed line is the case of $|q_a B|/\kappa^2 = 1/4$.

and $\epsilon(\mu_a) = 2\epsilon(B)\eta_a(\nu_a - 1/2)$. The asymptotic behavior of $|\mu_a|$ is obtained from (13). We can easily see that $|\mu_a| \simeq \omega_1^a/2 = \sqrt{|q_a B|/(2q_a^2)}$ in the low temperature limit ($\beta \to \infty$). At high temperature, we also see that $|\mu_a|$ behaves like $\sim \beta|B|$. We show the behavior of the chemical potential as a function of T at some values of B (Figure 1). It can be seen in the figure that $|\mu_a|$ is a monotonously decreasing (increasing) function of T for fixed $B > 0$ ($B < 0$) and is also a increasing function of $|B|$ with fixed T. We also find from the relation (13) that for small $\beta^2|B|$,

$$|\mu_a| \simeq \frac{1}{4\ln 2}\beta|B| + \mathcal{O}(|B|^2). \tag{14}$$

Note that the non-zero μ_a here comes from the asymmetry of the lowest Landau levels. In the case of $B = 0$, the asymmetry vanishes so that μ_a becomes zero.

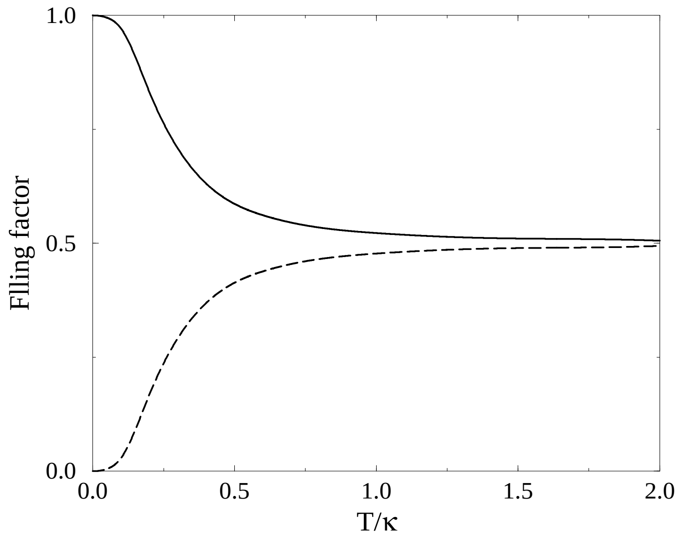

Figure 2: The thermal behavior of the filling factors $\langle\hat{\nu}^{\pm}\rangle$ for fixed $|B|$. The solid line shows the behavior of $\langle\hat{\nu}^{+}\rangle$. The dashed line is that of $\langle\hat{\nu}^{-}\rangle$. We here set $\nu^{+} = 1$ and $\nu^{-} = 0$ at $T = 0$, taking account of eq. (9).

Now we comment on the thermal behavior of the filling factors. The choice of the filling factors at zero temperature is taken to satisfy the consistency condition (9) with $\kappa \neq 0$. The thermal average of filling factors is easily calculated by using (8) as

$$\langle\hat{\nu}_a\rangle = \frac{1}{e^{-\beta q_a \mu_a \eta_a \epsilon(B)} + 1}. \tag{15}$$

When we set ν^{+} and ν^{-} (the filling factors for η_{+} and η_{-}-fermions, respectively) into 1 and 0 at $T = 0$ respectively, the thermal average $\langle\hat{\nu}^{+}\rangle$ ($\langle\hat{\nu}^{-}\rangle$) becomes the monotonous decreasing (increasing) function of T and $\langle\nu^{\pm}\rangle \to 1/2$ at large T (see Figure 2). This merely shows that the probability that the $\eta_{\pm}$-fermion exits in a lowest Landau level is multiplied by no statistical weight at high temperature limit.

3 Spontaneous Magnetization, Strategy to Finite Temperature System

In this section, we will show our strategy to study the spontaneous magnetization in the finite temperature system. To this aim, we start from a short review for the zero temperature case, which have been discussed at first by Hosotani [7, 8]. Next we shall discuss the extension to the finite temperature case.

At zero temperature, Hosotani has shown in the same model that the spontaneous magnetic field can be stable and then triggers the spontaneous breakdown of the Lorentz invariance. The effective potential has been calculated in a weak field approximation and the fermion massless limit [7]. The classical part ($\Delta\mathcal{E}^{(3)}$ in ref. [7]) of the potential yields the contributions of the orders of $|B|^{3/2}$ (matter) and $|B|^2$ (photon) with positive coefficients. The minimum of the potential of the classical part is then at $B = 0$. The radiative correction to the potential drastically changes the situation. The deviation of the fermion-loop contribution to the effective potential between $B \neq 0$ and $B = 0$, $\mathcal{E}^{\rm rad}$ ($\equiv \Delta\mathcal{E}^{(1)} - \Delta\mathcal{E}^{(2)}$, in ref. [7]), is given by

$$\mathcal{E}^{\rm rad} = -\frac{i}{2}\int \frac{d^3p}{(2\pi)^3} \ln \frac{(1+\Pi_0)\left\{1+(p_0^2\Pi_0 - \vec{p}^2\Pi_2)/p^2\right\} - (\kappa - \Pi_1)^2/p^2}{(B\to 0)}, \quad (16)$$

where $\Pi_i (= \sum_a \Pi_i^a), (i = 0, 1, 2)$ are defined from the one-loop contribution of the vacuum polarization tensor:

$$\begin{aligned}\Gamma^{\mu\nu}(p) &= (p^\mu p^\nu - p^2 g^{\mu\nu})\Pi_0(p) + i\epsilon^{\mu\nu\rho}p_\rho \Pi_1(p) \\ &\quad +(1-\delta^{\mu 0})(1-\delta^{\nu 0})(p^\mu p^\nu - \vec{p}^2\delta^{\mu\nu})(\Pi_2(p) - \Pi_0(p)). \quad (17)\end{aligned}$$

In the case of $\forall q_a = e(>0)$ and $N_f^+ = N_f^- \equiv N$ (called the chirally symmetric case), putting the filling factors $\nu^+ = 1$ and $\nu^- = 0$ on taking account of the condition (9), $\mathcal{E}^{\rm rad}$ is then calculated in the fermion one-loop level by taking a weak field approximation and the fermion massless limit: $\mathcal{E}^{\rm rad} = -e\kappa/\pi^2 \tan^{-1}(4/\pi)\,|B| + \cdots$. This implies that there cannot be the minimum value of the effective potential at the point of $B = 0$ so that the spontaneous magnetization occurs. Then by the relation $\langle 0|[iM^{0i}, F^{0j}(x)]|0\rangle = \epsilon^{ij}B \neq 0$ where $M^{\mu\nu}$ are the generators of the Lorentz transformation, it is found that the non-vanishing magnetic field B induces the spontaneous breakdown of

the Lorentz invariance. Note that the essential point for this result is the existence of the non-vanishing negative coefficient of the linear $|B|$ term in the effective potential, which comes from the radiative correction part $\mathcal{E}^{\mathrm{rad}}$. If we set the bare C-S parameter κ into zero, the linear term vanishes and the symmetry is restored.

From now, we discuss the extension of the effective potential to the finite temperature system. The effective potential (energy density), $\mathcal{E}(B)$, is then replaced by the thermodynamical potential $\Omega(B, T, \mu)$ [20]. In the case of $B = 0$, Bralic *et. al.* has calculated the potential in this model in the different context [11]. In the $B \neq 0$ case, the classical part of the fermion sector is given by

$$\Omega_0^{\mathrm{fermi}} = -\frac{1}{V\beta} \ln Z, \tag{18}$$

where V is the two-dimensional volume and Z is defined in eq. (10). We show an expression of $\Omega_0^{\mathrm{fermi}}$ in Appendix 1. We confirm that there is no linear B term in $\Omega_0^{\mathrm{fermi}}$ there [26]. The quantum fluctuation part of Ω is more important here because we are interested in the spontaneous magnetic field for which the quantum part is essential in the case of $T = 0$. The extension of this part to the finite temperature system is well performed by the Matsubara method [21]. The fluctuation interaction part, Ω^{rad}, of the thermodynamical potential is then obtained from the eq. (16) just by taking the replacement [22]

$$p^0 \rightarrow ip_3 = i\frac{2m\pi}{\beta},\ (m : \mathrm{integer}), \tag{19}$$

$$\int \frac{dp^0}{2\pi} \rightarrow \frac{i}{\beta}\sum_m . \tag{20}$$

and $\Pi_i^a(p^2, B) \rightarrow \tilde{\Pi}_i^a(\vec{p}^2, B, \beta)_m,\ (i = 0, 1, 2)$ for bosonic loop integrations. As to the fermion loops included in $\tilde{\Pi}_i$, we employ the replacement with imposing anti-periodic boundary condition,

$$k^0 \rightarrow ik_3 = i\frac{(2n+1)\pi}{\beta} + q_a\mu_a,\ (n : \mathrm{integer}), \tag{21}$$

$$\int \frac{dk^0}{2\pi} \rightarrow \frac{i}{\beta}\sum_n . \tag{22}$$

Thus the deviation of the quantum fluctuation part of the potential is formally expressed in the finite temperature system by

$$
\begin{aligned}
\Omega^{\mathrm{rad}} &= \frac{1}{2\beta}\sum_m \int \frac{d^2\vec{p}}{(2\pi)^2} \\
\times \;\ln &\; \frac{(1+\tilde{\Pi}_0)\left\{1+(p_3^2\tilde{\Pi}_0+\vec{p}^2\tilde{\Pi}_2)/(p_3^2+\vec{p}^{\,2})\right\}+(\kappa-\tilde{\Pi}_1)^2/(p_3^2+\vec{p}^{\,2})}{(B\to 0)}.
\end{aligned}
\tag{23}
$$

We, however, will not treat this expression directly. Since the non-zero coefficient of the linear B term is essential for the spontaneous magnetization, what we have only to know in detail for our purpose is just the coefficient $C(\beta,\mu_0)$ in Ω^{rad},

$$
\Omega^{\mathrm{rad}} = C(\beta,\mu_0)|B| + (\text{higher orders of } |B|), \tag{24}
$$

where $\mu_0 \equiv \mu(\beta)|_{B=0}$ is zero in the fermion massless limit because of eq. (14). We will concentrate our attention into $C(\beta)\equiv C(\beta,0)$ from now. The coefficient is decomposed into two parts as

$$
\begin{aligned}
C(\beta) &= \left.\frac{\partial\Omega^{\mathrm{rad}}}{\partial B}\right|_{B=0,\mu=0} + \left.\frac{\partial\Omega^{\mathrm{rad}}}{\partial\mu_a}\frac{\partial\mu_a}{\partial B}\right|_{B=0,\mu=0} \\
&\equiv C_0(\beta)+C_\mu(\beta).
\end{aligned}
\tag{25}
$$

The first term is obtained from eq. (23) as

$$
\begin{aligned}
C_0(\beta) &= \frac{1}{2\beta}\sum_{m=-\infty}^{\infty}\int\frac{d^2\vec{p}}{(2\pi)^2}\left[\frac{\partial\tilde{\Pi}_0}{\partial B}\left\{p_3^2+\vec{p}^{\,2}+(p_3^2\tilde{\Pi}_0+\vec{p}^{\,2}\tilde{\Pi}_2)\right\}\right. \\
&\quad \left. +(1+\tilde{\Pi}_0)\left\{p_3^2\frac{\partial\tilde{\Pi}_0}{\partial B}+\vec{p}^{\,2}\frac{\partial\tilde{\Pi}_2}{\partial B}\right\}-2(\kappa-\tilde{\Pi}_1)\frac{\partial\tilde{\Pi}_1}{\partial B}\right] \\
&\quad \times\left[\left(1+\tilde{\Pi}_0\right)\left\{(p_3^2+\vec{p}^2)+\right.\right. \\
&\qquad \left.\left.\left. p_3^2\tilde{\Pi}_0+\vec{p}^2\tilde{\Pi}_2\right\}+(\kappa-\tilde{\Pi}_1)^2\right]^{-1}\right|_{B=0,\mu=0}.
\end{aligned}
\tag{26}
$$

The second term in eq. (25) is the contribution of the chemical potential μ. We however easily see that $C_\mu(\beta)$ does not contribute. Since Ω^{rad} is the

deviation of the quantum fluctuation between at $B \neq 0$ and at $B = 0$, we can easily see that $C_\mu(\beta)$ becomes zero by virtue of eq. (13).

As long as the coefficient $C(\beta)$ is negative, the spontaneous magnetic field does not vanish. In order to evaluate the behavior of $C(\beta)$ by using (26), we need to calculate $\tilde{\Pi}_i$'s. In the next section, we will show the details of the calculation of $\tilde{\Pi}_i$'s. Since we are interested in the coefficient of the linear $|B|$ term, $C_0(\beta)$, the calculation will be performed up to the order $\mathcal{O}(B^1)$ there.

4 Boson Self-energies at finite temperature

This section will be devoted to the calculation of the boson self-energies $\tilde{\Pi}_i (i = 0, 1, 2)$ at $T \neq 0$, which are composed of the fermion one-loop diagrams. In the $T = 0$ field theory, the fermion propagator $S(x, y)$ in the classical field B is expressed by the proper-time method [7]. We just quote the expressions and some character of $S(x, y)$ in Appendix 2. The fermion one-loop contributions to the vacuum polarization tensors are then calculated by

$$\Gamma^{a\mu\nu}(p) = i\, q_a^2 \int \frac{d^3k}{(2\pi)^3} \mathrm{tr}\left[\gamma^\mu S_0^a(k) \gamma^\nu S_0^a(k-p)\right], \tag{27}$$

where propagator $S_0^a(p)$ is defined in eq. (A.7) and $\Gamma^{\mu\nu}(= \sum_a \Gamma^{a\mu\nu})$ are related to $\Pi_i(= \sum_a \Pi_i^a)$ in eq. (17).

Since we are interested in the coefficient of $|B|$ in Ω^{rad}, we here take the weak field approximation up to the order $\mathcal{O}(B^1)$ in the calculation with setting $\mu_a = 0$. (In Sec. 6, we will adopt a little better approximation to evaluate the spectrum of the photons.) We here show it only in the case of $\eta^+ 1$-fermion in $q_a B > 0$ because other cases are connected with this case by eqs. (A.6) and (A.9). At first, we consider the contributions of fermions with $\nu_a = 0$. We drop the fermion index a for a while just for avoiding complexity. The expansion of $S_0(p)^{\eta=+1}_{\nu=0}$ by $1/l^2 = |qB|$ becomes

$$\begin{aligned} S_0(p)^{\eta=+1}_{\nu=0} &= \frac{m + \not{p}}{p^2 - m^2 + i\epsilon} - \frac{m\sigma_3 + p_0 I}{(p^2 - m^2 + i\epsilon)^2} l^{-2} + \mathcal{O}(l^{-4}) \\ &\equiv S_0(p)^{\eta=+1(0)}_{\nu=0} + S_0(p)^{\eta=+1(1)}_{\nu=0} l^{-2} + \mathcal{O}(l^{-4}). \end{aligned} \tag{28}$$

We then obtain the expansion of $\Gamma^{\mu\nu}$ from eqs. (27) and (28) in the case of

$\nu = 0$: $\Gamma^{\mu\nu}(p)^{\eta=+1}_{\nu=0} = \Gamma^{\mu\nu}(p)^{\eta=+1(0)}_{\nu=0} + \Gamma^{\mu\nu}(p)^{\eta=+1(1)}_{\nu=0} l^{-2} + \mathcal{O}(l^{-4})$, where

$$\Gamma^{\mu\nu}(p)^{\eta=+1(0)}_{\nu=0} \equiv i\,q^2 \int \frac{d^3k}{(2\pi)^3} \mathrm{tr}\left[\gamma^\mu S_0(k)^{(0)}_{\nu=0} \gamma^\nu S_0(k-p)^{(0)}_{\nu=0}\right], \tag{29}$$

$$\Gamma^{\mu\nu}(p)^{\eta=+1(1)}_{\nu=0} \equiv i\,q^2 \int \frac{d^3k}{(2\pi)^3} \left\{\mathrm{tr}\left[\gamma^\mu S_0(k)^{(0)}_{\nu=0} \gamma^\nu S_0(k-p)^{(1)}_{\nu=0}\right] + \mathrm{tr}\left[\gamma^\mu S_0(k)^{(1)}_{\nu=0} \gamma^\nu S_0(k-p)^{(0)}_{\nu=0}\right]\right\}. \tag{30}$$

Since we can easily check $\Gamma^{\mu\nu}(p)^{\eta=+1(1)}_{\nu=0}$ to be zero due to the properties of the Dirac γ-matrices, we can regard $\Gamma^{\mu\nu}(p)^{\eta=+1}_{\nu=0}$ as $\Gamma^{\mu\nu}(p)^{\eta=+1(0)}_{\nu=0}$ in this approximation. Next, let us consider the case of $\nu = 1$. The propagator in this case is given in (A.11). It is convenient to see the deviation $\delta\Gamma^{\mu\nu}(p)^{\eta=+1} \equiv \Gamma^{\mu\nu}(p)^{\eta=+1}_{\nu=1} - \Gamma^{\mu\nu}(p)^{\eta=+1}_{\nu=0}$, where

$$\begin{aligned}\delta\Gamma^{\mu\nu}(p)^{\eta=+1} &= i\,q^2 \int \frac{d^3k}{(2\pi)^3} \left\{\mathrm{tr}\left[\gamma^\mu S_0(k)^{\eta=+1}_{\nu=0} \gamma^\nu f(k-p)\right] + \mathrm{tr}\left[\gamma^\mu f(k) \gamma^\nu S_0(k-p)^{\eta=+1}_{\nu=0}\right]\right\} \\ &\quad + i\,q^2 \int \frac{d^3k}{(2\pi)^3} \mathrm{tr}\left[\gamma^\mu f(k) \gamma^\nu f(k-p)\right].\end{aligned} \tag{31}$$

Since $f(p)$ includes the part $e^{-\vec{p}^{\,2} l^2}$ (see Appendix 2), it cannot be taken the Taylor expansion by $1/l^2$ in a usual sense. We have to keep this part in our calculation whereas we drop terms with the part $l^{-2} \times e^{-\vec{p}^{\,2} l^2}$. The last term of r.h.s. in eq. (31) is then dropped in this sense. Hence, in our approximation, $\tilde{\Pi}_i$'s are calculated by the following expressions:

$$\Gamma^{\mu\nu}(p)^{\eta=+1}_{\nu=0} = i\,q^2 \int \frac{d^3k}{(2\pi)^3} \mathrm{tr}\left[\gamma^\mu S_0(k)^{(0)}_{\nu=0} \gamma^\nu S_0(k-p)^{(0)}_{\nu=0}\right] + \mathcal{O}(l^{-4}), \tag{32}$$

$$\begin{aligned}8\Gamma^{\mu\nu}(p)^{\eta=+1}_{\nu=1} &= \Gamma^{\mu\nu}(p)^{\eta=+1}_{\nu=0} + i\,q^2 \int \frac{d^3k}{(2\pi)^3} \left\{\mathrm{tr}\left[\gamma^\mu S_0(k)^{\eta=+1(0)}_{\nu=0} \gamma^\nu f(k-p)\right]\right. \\ &\quad \left. +\ \mathrm{tr}\left[\gamma^\mu f(k) \gamma^\nu S_0(k-p)^{\eta=+1(0)}_{\nu=0}\right]\right\} + \mathcal{O}(l^{-2} e^{-l^2 p^2}).\end{aligned} \tag{33}$$

Before proceeding to the calculation of $\tilde{\Pi}_i$, we mention the consistency of our approximation scheme. The self-energies at $T = 0$ (Π_i's) are calculated by using the expressions in eqs. (32) and (33) up to the order $\mathcal{O}(B^1)$. They

are presented in eqs. (A.12) $\sim$ (A.17) in Appendix 3. We can see that these expressions are consistent with the calculation by Hosotani [7]. His calculation of the self-energies by virtue of the proper-time representation includes the ultra-violet divergence which has to be renormalized. On the other hand, we do not have any ultra-violet divergence in our calculation. This is because the divergence part appears only in the term higher order than $\mathcal{O}(|B|)$, which we are now neglecting.

Now we go to the case in the finite temperature system. By applying the replacements (21) and (22) (and $p_0 \to ip_3$) to Π_i's calculated in eqs. (A.12) $\sim$ (A.17) with eq. (A.34), we obtain $\tilde{\Pi}_0$, $\tilde{\Pi}_1$ and $\tilde{\Pi}_2$ in the weak field approximation, which are presented in Appendix 4. From now we take the fermion massless limit ($m_a \to 0$). We also restore the fermion index a in the expression. For the case of $\nu_a = 0$, $\tilde{\Pi}_i$'s become then

$$
\begin{aligned}
\tilde{\Pi}^a_{0,\nu=0}(p_3, \vec{p}^{\,2}, B, \beta) &= \frac{q_a^2}{16} \frac{1}{(\vec{p}^{\,2} + p_3^2)^{1/2}} + A_0^a(p_3, |\vec{p}|, \beta) + \mathcal{O}(|B|^2), \quad (34) \\
\tilde{\Pi}^a_{1,\nu=0}(p_3, \vec{p}^{\,2}, B, \beta) &= 0 + \mathcal{O}(|B|^2), \quad (35) \\
\tilde{\Pi}^a_{2,\nu=0}(p_3, \vec{p}^{\,2}, B, \beta) &= -\frac{p_3^2}{\vec{p}^{\,2}} \tilde{\Pi}^a_{0,\nu=0}(p_3, \vec{p}^{\,2}, B, \beta) \\
&\quad + \frac{q_a^2}{16\vec{p}^{\,2}} (\vec{p}^{\,2} + p_3^2)^{1/2} - \frac{1}{\vec{p}^{\,2}} A_2^a(p_3, |\vec{p}|, \beta) + \mathcal{O}(|B|^2), \quad (36)
\end{aligned}
$$

where A_0^a and A_2^a are contributions of finite temperature effects from the fermion loops and take the forms as

$$
\begin{aligned}
A_0^a(p_3, |\vec{p}|, \beta) &= \frac{q_a^2}{2\pi \vec{p}^{\,2}} \int_0^\infty dk \frac{1}{e^{\beta k} + 1} \\
&\quad \times \left\{ 1 - \left[\frac{\sqrt{(\vec{p}^{\,2} + p_3^2 - 4k^2)^2 + 16k^2 p_3^2} + \vec{p}^{\,2} + p_3^2 - 4k^2}{2(\vec{p}^{\,2} + p_3^2)} \right]^{\frac{1}{2}} \right\}, \quad (37) \\
A_2^a(p_3, |\vec{p}|, \beta) &= \frac{p_3^2 q_a^2}{2\pi \vec{p}^{\,2}} \int_0^\infty dk \frac{1}{e^{\beta k} + 1} \\
&\quad + \frac{q_a^2}{4\pi} \frac{(\vec{p}^{\,2} + p_3^2)^{1/2}}{\vec{p}^{\,2}} \int_0^\infty dk \frac{1}{e^{\beta k} + 1} \\
&\quad \times \frac{(*)}{\sqrt{(\vec{p}^{\,2} + p_3^2 - 4k^2)^2 + 16k^2 p_3^2}}, \quad (38)
\end{aligned}
$$

where

$$(*) \equiv \sqrt{2}\left\{(4k^2-p_3^2)\left[\sqrt{(\vec{p}^{\,2}+p_3^2-4k^2)^2+16k^2p_3^2}+\vec{p}^{\,2}+p_3^2-4k^2\right]^{1/2}\right.$$
$$\left.-4k(p_3^2)^{1/2}\left[\sqrt{(\vec{p}^{\,2}+p_3^2-4k^2)^2+16k^2p_3^2}-(\vec{p}^{\,2}+p_3^2-4k^2)\right]^{1/2}\right\}.$$

We can see from eqs. (37) and (38) that $\tilde{\Pi}_0^a$ and $\tilde{\Pi}_2^a$ grow like $\sim T$ at high temperature for fixed p_3. This behavior has already been pointed out in ref. [12], where the calculation has been based on the real-time formalism. As to the case $\nu=1$, we obtain

$$\tilde{\Pi}^a_{0,\nu=1}(p_3,\vec{p}^{\,2},B,\beta) = \tilde{\Pi}^a_{0,\nu=0}(p_3,\vec{p}^{\,2},B,\beta), \tag{39}$$

$$\tilde{\Pi}^a_{1,\nu=1}(p_3,\vec{p}^{\,2},B,\beta) = \eta_a\frac{q_a^3}{\pi}\frac{1}{\vec{p}^{\,2}+p_3^2}|B|+\mathcal{O}(e^{-\vec{p}^{\,2}/(q_a|B|)}), \tag{40}$$

$$\tilde{\Pi}^a_{2,\nu=1}(p_3,\vec{p}^{\,2},B,\beta) = \tilde{\Pi}^a_{2,\nu=0}(p_3,\vec{p}^{\,2},B,\beta). \tag{41}$$

Note that only $\tilde{\Pi}^a_{1,\nu=1}$ depends on the magnetic field in this approximation. We can neglect the term of the order $\mathcal{O}(e^{-\vec{p}^{\,2}/(q_a|B|)})$ in eq. (40) for the evaluation of $C(\beta)$ because this term is sufficiently smaller than the linear term for small $|B|$.

Therefore, we have calculated all the self-energies $\tilde{\Pi}_i$ up to the order $\mathcal{O}(B^1)$ in the fermion massless limit. In the next section, we will investigate the character of the effective potential $\Omega^{\rm rad}$ by analyzing the coefficient of the linear term $C(\beta)$ by using the results in (34) $\sim$ (41).

5 Thermodynamical Potential

We here examine the effective potential in the finite temperature system. The purpose is, as we mentioned before, to decide whether the spontaneous magnetic field, that takes non-zero value at $T=0$, vanishes at high temperature or not. We shall analyze the coefficient of the linear $|B|$ term $C_0(\beta)$ in the potential $\Omega(B,\beta,\mu_a)$ by using $\tilde{\Pi}_i$'s calculated in the massless limit in the last section. In the evaluation, we set $q_a=e>0$, and assume $N_f^+=N_f^-=N$. The filling factors ν_+ and ν_- are set their value into 1 and 0 at $T=0$, respectively. The condition (9) becomes then $\kappa=e^2N/(2\pi)$. By using eqs. (34) $\sim$

(41) as well as this condition, the coefficient $C_0(\beta)$ in eq. (26) is expressed as

$$C_0(\beta) = -\frac{e\kappa}{2\pi^2}\alpha \sum_{m=-\infty}^{\infty} \int_0^\infty dx \frac{x^5}{F_m(x\,,\,\alpha)}, \tag{42}$$

where $\alpha \equiv 2\pi T/\kappa$ and $x \equiv |\vec{p}|/\kappa$.

The function $F_m(x\,,\,\alpha)$ in r.h.s. of eq. (42) is defined as

$$\begin{aligned} F_m(x\,,\,\alpha) \;\equiv\; & \left[x^2\left(x^2+(\alpha m)^2\right)+\frac{\pi}{4}x^2\sqrt{x^2+(\alpha m)^2}+\tilde{A}_0(x,\,\alpha,\,m)\right] \\ & \times\left[x^2\left(x^2+(\alpha m)^2\right)+\frac{\pi}{4}x^2\sqrt{x^2+(\alpha m)^2}-\tilde{A}_2(x,\,\alpha,\,m)\right] \\ & +x^4\left(x^2+(\alpha m)^2\right)\,, \end{aligned} \tag{43}$$

where

$$\tilde{A}_0(x,\alpha,m) \;\equiv\; \sum_a A_0^a\left(\kappa\alpha m, \kappa x, 2\pi/(\kappa\alpha)\right) x^2\left(x^2+(\alpha m)^2\right), \tag{44}$$

$$\tilde{A}_2(x,\alpha,\,m) \;\equiv\; \sum_a A_2^a\left(\kappa\alpha m, \kappa x, 2\pi/(\kappa\alpha)\right)\left(\frac{x}{\kappa}\right)^2. \tag{45}$$

Before the numerical analyses, let us see the asymptotic behavior of $C_0(\beta)$ described in (42). At first, we consider the low temperature limit. Since $\tilde{A}_0$ and $\tilde{A}_2$ have the factor $1/(e^{\beta k}+1)$ (see eqs. (37) and (38)), these terms are rapidly dumped. Then $F_m(x\,,\,\alpha)$ is replaced by $x^4 \times f(x\,,\,\alpha m)$, where

$$f(x\,,\,\alpha m) = \left(x^2+(\alpha m)^2\right)\left\{1+\left(\frac{\pi}{4}+\sqrt{x^2+(\alpha m)^2}\right)^2\right\}. \tag{46}$$

The coefficient is then calculated by making use of the Euler-Mclaurin's mathematical formula (see Appendix 6.) as below,

$$\begin{aligned} C_0(\beta) \;\to\; & -\frac{e\kappa}{2\pi^2}\alpha\left\{2\sum_{m=0}^{\infty}\int_0^\infty dx\frac{x}{f(x,\alpha m)}-\int_0^\infty dx\frac{x}{f(x,0)}\right\} \\ = \; & -\frac{e\kappa}{2\pi^2}\alpha\left\{\frac{2}{\alpha}\int_0^\infty dy\int_0^\infty dx\frac{x}{f(x,y)}+\lim_{m\to\infty}\int_0^\infty dx\frac{x}{f(x,\alpha m)}+\mathcal{O}(\alpha)\right\} \\ = \; & -\frac{e\kappa}{\pi^2}\int_0^\infty dr\frac{1}{1+\left(\frac{\pi}{4}+r\right)^2}+\mathcal{O}(\alpha^2), \quad (x=r\cos\theta, y=r\sin\theta) \\ = \; & -\left\{\frac{e\kappa}{\pi^2}\tan^{-1}\frac{4}{\pi}+\mathcal{O}(T^2)\right\}, \quad (T\simeq 0)\,. \end{aligned} \tag{47}$$

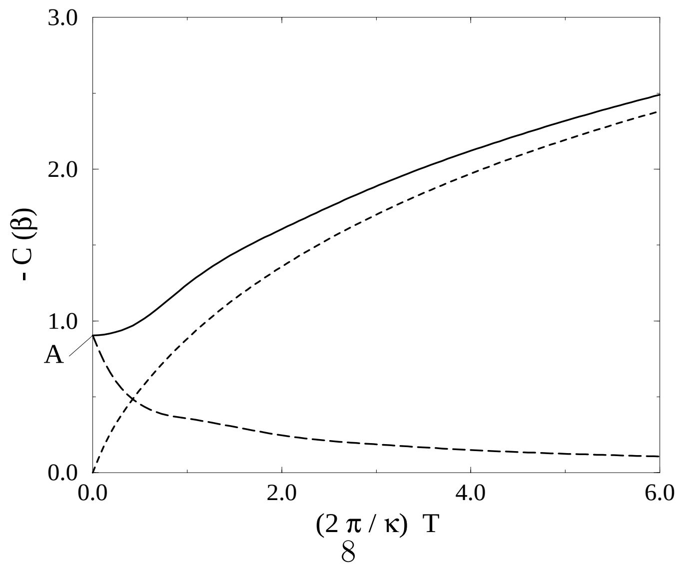

Figure 3: The behavior of the coefficient $C(\beta)$ ($\beta = 1/T$) is shown by the solid line. It can be seen that the coefficient is negative and does not vanish even at high temperature. Rather, the absolute value becomes larger as $\sim \log T$. The point A which is the low temperature limit takes the value $\kappa e/\pi^2 \tan^{-1}(4/\pi)$. The dotted line is the contribution of the zero-mode. The dashed line is the summation of the other modes.

Thus we indeed reproduce the result in [7]. Secondly, at high temperature region, the contribution of the zero mode ($m = 0$) becomes dominant because that of the non-zero modes ($m \neq 0$) is relatively suppressed by $1/T$ as

$$\begin{aligned} C_0(\beta)_{m\neq 0\text{modes}} &= -\frac{e\kappa}{\pi^2}\alpha \sum_{m=1}^{\infty} \int_0^{\infty} dx \frac{x^5}{F_m(x,\alpha)} \\ &\to -\frac{\kappa e}{6\alpha}\int_0^1 dt \frac{\sinh t}{\cosh^3 t}, \quad (x = \alpha m \sinh t) \\ &= -\frac{\kappa^2 e}{24\pi}\frac{1}{T}, \quad (T \to \infty). \end{aligned} \tag{48}$$

The contribution of the zero mode does not vanish in the limit and behaves

like

$$C_0(\beta)_{\rm m=0mode} \to -\frac{e\kappa}{2\pi \ln 2} \ln \frac{T}{\kappa}, \quad (T \to \infty). \tag{49}$$

Eqs. (48) and (49) show that $C_0(\beta)$ does not vanish at any finite temperature. The numerical analyses show that the coefficient $C_0(\beta)$ is always negative at any temperature (Figure 3). The non-vanishing $C_0(\beta)$ means that the thermodynamical potential Ω does not have its minimum value at $B = 0$ at any temperature.

6 Photon Spectrum at Finite T and μ

At $T = 0$ and $\mu = 0$ case the spontaneous magnetic field implies the dynamical Lorentz symmetry breaking. The massless pole then appears in the spectrum of the physical photons by virtue of the Nambu-Goldstone theorem. Namely, there is no spontaneous magnetization at tree level and photons then have a topological mass $|\kappa|$ whereas the magnetization occurs at one loop level and photons then become massless. The massless-ness comes from the exact cancellation of the bare topological mass κ by $\Pi_1(0)$ [7, 8, 15]. It has been well known that the cancellation is exact for all orders of the perturbation [14].

In the finite temperature system, the Lorentz symmetry is explicitly broken by the temperature effects. On the other hand, we have just shown by the thermodynamical potential approach that the spontaneous magnetic field does not vanish at any finite temperature. It is of very interest to examine the photon spectrum in the finite temperature system. The photons then becomes massive by temperature effects even if the topological mass is cancelled out at loop levels. From the Lagrangian (1) the photon spectrum in the finite temperature system is determined by [12, 18]

$$(p^2 - \tilde{\Pi}_L^2)(p^2 - \tilde{\Pi}_T^2) - p^2 \tilde{\Pi}_{\rm topo}^2 = 0, \tag{50}$$

where $\tilde{\Pi}_L$ and $\tilde{\Pi}_T$ becomes the thermal masses for the longitudinal and transverse modes respectively when the topological mass $\tilde{\Pi}_{\rm topo}$ is zero. In our model $\tilde{\Pi}_{L,T}$ and $\tilde{\Pi}_{\rm topo}$ are

$$\tilde{\Pi}_L(p^2) \;\; = \;\; -p^2 \sum_a \tilde{\Pi}_0^a(p^2), \tag{51}$$

$$\tilde{\Pi}_T(p^2) = -p_0^2\sum_a \tilde{\Pi}_0^a(p^2) + \vec{p}^{\,2}\sum_a \tilde{\Pi}_2^a(p^2), \quad (52)$$

$$\tilde{\Pi}_{\rm topo}(p^2) = \kappa - \sum_a \tilde{\Pi}_1^a(p^2), \quad (53)$$

where we wrote $p_3 = -ip_0$. By using eqs. (34), (36), (39) and (41), the expressions of $\tilde{\Pi}_L$ and $\tilde{\Pi}_T$ at high temperature are obtained in the chirally symmetric case as

$$\tilde{\Pi}_L = 4N\omega_p^2\frac{p^2}{\vec{p}^{\,2}}\left(\frac{p_0}{p} - 1\right), \quad (54)$$

$$\tilde{\Pi}_T = 4N\omega_p^2\frac{p_0}{\vec{p}^{\,2}}\left(p_0 - p\right), \quad (55)$$

where $\omega_p^2 \equiv (e^2\ln 2/4\pi)T$ is the plasma frequency. These are consistent with the results by Klein-Kreisler *et. al.* [12].

At first, let us consider the case of $\tilde{\Pi}_{\rm topo}(0) = 0$. In terms of the field strength, $\tilde{\Pi}_L$ and $\tilde{\Pi}_T$ are the thermal masses of electric fields and magnetic fields respectively. In the static limit ($p_3 = 0, \vec{p} \to 0$) [17], we easily obtain $\tilde{\Pi}_L = 4N\omega_p$ and $\tilde{\Pi}_T = 0$. The electric fields are then massive and become short-range force and thus screened, whereas the magnetic fields are massless and remains long-range forces. In dynamic cases such as plasma oscillation, both fields are massive with the same mass $\sim \omega_p$. Secondly, in the case of $\tilde{\Pi}_{\rm topo}(0) \equiv \tilde{\kappa} \neq 0$, the electric and magnetic fields become both massive. They have the same static masses $\sim \sqrt{4N\omega_p^2 + \tilde{\kappa^2}}$. Also, the dynamic masses of the photons become approximately $\sim \sqrt{2N\omega_p^2 + \kappa^2}$. Therefore if $\tilde{\kappa}$ is zero at any temperature, we can obtain a certain relationship at finite temperature between the magnetization and the statement that the magnetic fields remain long-range forces.

In the following we shall show that in our model, the topological mass $\tilde{\kappa}$ vanishes at any temperature:

$$\tilde{\kappa} = \kappa - \sum_a \tilde{\Pi}_1^a(0) = 0. \quad (56)$$

We stress that this is not contradictory with the results by Lykken *et. al.* [24]. In our model, we at first put the fermion distribution at zero temperature. The chemical potential is then not a free parameter but a function of B and T, which is determined by the consistency condition at finite temperature.

Now we calculate $\tilde{\Pi}_1(0)$ in the weak field approximation. we here leave the fermion massive (but not so heavy). For the $\nu=0$ fermions, the contribution is easily calculated from (A.25) by putting $p_3, \vec{p}=0$ and using the formula (A.34) as

$$\tilde{\Pi}_1^{a\nu=0}(0) = -\frac{q_a^2\eta_a}{4\pi}\left(1 - \frac{1}{e^{\beta(m_a - q_a\mu_a)}+1} - \frac{1}{e^{\beta(m_a + q_a\mu_a)}+1}\right). \tag{57}$$

We can easily see that, when $m_a = 0$, $\tilde{\Pi}_1^{a\nu=0}(0)$ becomes zero at any temperature and density as in eq. (35). In the low temperature limit, eq. (57) reproduces the result in [5, 7]. At high temperature limit, it becomes zero. Next, we consider the case of $\nu=1$ fermions. We cannot use eq. (40) to take the limit of $p_3, \vec{p}=0$ because the approximation for eq. (40) is of the order $\mathcal{O}(|B|)$ which is not appropriate here. So we go back to the expression in eq. (A.16). Here we consider $\tilde{\Pi}_1^{a\nu=1}(0)$ as the form getting from applying the replacement (19), (21) and (22) to eq. (A.16). By using the formula (A.34), we can decompose $\delta\tilde{\Pi}_1^a (\equiv \tilde{\Pi}_1^{a\nu=1} - \tilde{\Pi}_1^{a\nu=0})$ as

$$\delta\tilde{\Pi}_1^a(p) = \delta\Pi_1^a(p) + \tilde{\Pi}_1^a(p,\beta)_{\text{Thermo}}, \tag{58}$$

where $\delta\Pi_1^a$ is the deviation between $\nu=0$ and $\nu=1$ of the induced C-S term at $T=0$, which has been calculated by using the proper time representation as [7],

$$\delta\Pi_1^a(p) = \frac{\eta_a q_a^2}{2\pi}\left(\frac{1}{2 - p^2 l_a^2 - 2m_a p_0 l_a^2} + \frac{1}{2 - p^2 l_a^2 + 2m_a p_0 l_a^2}\right). \tag{59}$$

In eq. (59), when we neglect the contribution of higher orders of $|B|$ than the first order, the result in eq. (A.22) is of course reproduced. This becomes $\eta_a q_a^2/(2\pi)$ for $p_\mu = 0$. The second term in r.h.s. in eq. (58) is defined in eq (A.30). We obtain the expression as (see Appendix 5.)

$$\begin{aligned}
&\tilde{\Pi}_1^a(p,\beta,B,\mu_a)_{\text{Thermo}} \\
&\quad = \eta_a q_a^2 l_a^2 p_0(p_0+2m_a) e^{-p_0(p_0+2m_a)l_a^2}\left\{\theta(p_0) - \theta(-p_0-2m_a)\right\} \\
&\quad\times\left\{\frac{\theta(-p_0-m_a+\mu_a)}{1+e^{\beta(-p_0-m_a+\mu_a)}} - \frac{\theta(p_0+m_a-\mu_a)}{1+e^{\beta(p_0+m_a-\mu_a)}} + \theta(p_0+m_a-\mu_a)\right\} \\
&\quad -\eta_a q_a^2 l_a^2 p_0(p_0-2m_a) e^{-l_a^2 p_0(p_0-2m_a)}\left\{\theta(-p_0) - \theta(p_0-2m_a)\right\} \\
&\quad\times\left\{\frac{\theta(m_a-\mu_a)}{1+e^{\beta(m_a-\mu_a)}} - \frac{\theta(-m_a+\mu_a)}{1+e^{\beta(-m_a+\mu_a)}} - \theta(m_a-\mu_a)\right\} + \mathcal{O}(|\vec{p}|).
\end{aligned} \tag{60}$$

It is easily seen from eq. (60) that the additional term $\tilde{\Pi}_1^{a\nu=1}(0)_{\rm Thermo}$ becomes zero at any temperature.

Thus, from eqs. (57), (58) and (59) with the condition (9), we find that eq. (56) holds when the fermion massless limit is taken or when the chirally symmetric case ($N_f^+ = N_f^- = N$) with massive (light) fermions is considered. In those cases, the following statement is allowed. According to the spontaneous magnetization, the magnetic part of the field strength has no any static mass and become long-range forces. If there is no spontaneous magnetization, the magnetic field becomes short range force due to the non-vanishing topological mass ($\tilde{\kappa} = \kappa \neq 0$).

7 Discussion

In Sec. 6 we have shown that the spontaneous magnetization induces the fact that the static masses of magnetic fields vanish and the magnetic force becomes a long-range one. In spite of the direct breakdown of the Lorentz invariance by temperature effects, this relation between the magnetization and massless-ness of the magnetic fields is quite interesting, though the connection with the Nambu-Goldstone theorem has not been clarified yet. In addition, eq. (56) is indeed consistent with the result from the thermodynamical potential approach.

Now let us consider the reason why the spontaneous magnetization occurs even at high temperature. This result might look somewhat strange when we remember various symmetries which are spontaneously broken at low temperature and restored at high temperature; chiral symmetry, gauge symmetry for weak interaction, some physics of magnetization like in ferromagnetic materials [23] and so on. At $T = 0$ case, the viewpoint from observing the shift in zero-point energies of photons could explain the appearance of the negative coefficient of the linear $|B|$ term to some extent [8]. Similar approximate estimation at finite temperature can be also useful. In the finite temperature system, the free energies of photons should be considered as well as the zero-point energies. If there is no magnetic fields, the photons have the dynamic mass $\sim \sqrt{2N\omega_p^2 + \kappa^2}$ [17]. On the other hand, if the spontaneous magnetization occurs, the mass is shifted to $\sim \sqrt{2N\omega_p^2}$. The difference of the free energies of the photons between these cases is expressed

as

$$
\begin{aligned}
\Delta F^{\text{boson}} &= F^{\text{boson}}(m^2 = 2N\omega_p^2) - F^{\text{boson}}(m^2 = 2N\omega_p^2 + \kappa^2) \\
&= \Delta E_{\text{z.p.}} + \Delta \tilde{F}^{\text{boson}},
\end{aligned} \tag{61}
$$

where $\Delta E_{\text{z.p.}}$ is deference of the photon zero-point energies and

$$
\Delta \tilde{F}^{\text{boson}} = \frac{1}{\beta} \int \frac{d^2\vec{p}^{\,2}}{(2\pi)^2} \ln \frac{1 - e^{-\beta\sqrt{\vec{p}^{\,2}+2N\omega_p^2}}}{1 - e^{-\beta\sqrt{\vec{p}^{\,2}+2N\omega_p^2+\kappa^2}}}. \tag{62}
$$

The rough estimation for $\Delta \tilde{F}^{\text{boson}}$ shows that there is the linear $|B|$ term with negative coefficients for small $|B|$ region, whereas the linear term in $\Delta E_{\text{z.p.}}$ vanishes at high temperature limit. On the other hand, it can be seen that the positive contribution of B fields to Ω_0^{fermi} amounts to the order $\mathcal{O}(|B|^2)$ in the fermion massless limit. (When fermions are massive, there is the linear term with positive coefficient at $T = 0$. [10, 25]) These estimations are summarized in Appendix 1. Therefore we can realize to some extent that the spontaneous magnetization occurs even at finite temperature at least at small B cases.

We have seen that the coefficient $C(\beta)$ becomes larger in the high temperature region (see fig. 3). We stress that this does not always mean that the spontaneous magnetic field, which is determined by observing the minimum of the potential, is an increasing function of T. Rather, it may be a decreasing function if the contribution of the higher orders of B to Ω^{rad} becomes dominant and it holds the value of the potential upper as temperature grows in the large B region. Then expectation value $\langle B \rangle$ would become smaller, even though $C(\beta)$ become larger as T grows. In this case, $\langle B \rangle$ goes to vanish effectively at high temperature limit. Such situation is very similar to the restoration of the parity violation analyzed by Klein-Kreisler *et.al.* [12]. We, however, do not discuss any more the contribution of higher order of B to the thermodynamical potential $\Omega(B, T)$ because our calculation here is basically due to the weak field approximation.

When we consider this model in a realistic system such as a planar system in 3+1 dimensional world, the spontaneous magnetic field keeps to take nonzero values up to a certain critical temperature where a kind of the hopping to 3+1 dimension would occur and the system would be no longer regarded as a planar system. These characteristics may be interesting in the connection to the physics such as the high T_C superconductivity [3].

8 Conclusion

In this paper, we have investigated the spontaneous magnetization in QED_3 with a bare C-S term in the finite temperature system. At first, we have analyzed the thermodynamical potential Ω as a function of temperature, chemical potential and magnetic field in the fermion one-loop level and the massless limit. Through these analyses, the consistency condition at finite temperature have been assumed, which gives the relation between μ, $|B|$ and β. We have found that the negative coefficient of the linear B term is induced in Ω by radiative correction. It means that the spontaneous magnetization can occur even at high temperature as well as at zero temperature. Secondly the spectrum of the physical photons has been investigated. At $T = 0$, the photons become massless by radiative effects so that they are regarded as the Nambu-Goldstone bosons for the spontaneous Lorentz symmetry breaking due to $B \neq 0$. We found that in our model, the topological mass vanish in the loop level even at finite temperature if the classical magnetic field exists. Since the magnetic force is a long-range one even at finite temperature unless there is a non-vanishing topological mass $\tilde{\Pi}_{\rm topo}$, we find that the following statement is allowed. According to the spontaneous magnetization, the magnetic part of the field strength has no any static mass and become long-range forces. This statement is consistent with the results by the thermodynamical potential approach above. Therefore we conclude that the spontaneous magnetic field, which is generated at $T = 0$, does not vanish even at finite temperature. This result is valid for the case of the fermion massless limit.

As to the case with massive fermions, we have found that $\tilde{\Pi}_{\rm topo}$ vanishes in the chirally symmetric case ($N^+ = N^- = N$) putting $q_a = e > 0$ and $m_a = m \neq 0$. However even in this case it remains unknown whether the spontaneous magnetization occurs at finite temperature in the model with massive fermions. We have not studied the details of the massive fermion cases by thermodynamical potential approach yet. This is a future problem. These cases would be more important in connection with the fractional quantum Hall effects and high T_C-superconductivity.

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APPENDICES

APPENDIX 1:

The Hamiltonian operator, $\hat{H} = \sum_a \hat{H}_a$, and the charge operator, $\hat{Q} = \sum_a \hat{Q}_a$, are given by

$$\hat{H}_a = \sum_{n=0}^{\infty}\sum_p a^\dagger_{np}a_{np}\omega^a_n + \sum_{n=1}^{\infty}\sum_p b^\dagger_{np}b_{np}\omega^a_n - \sum_{n=1}^{\infty}\sum_p \omega^a_n, \tag{A.1}$$

$$\hat{Q}_a = \left\{ \begin{array}{c} q_a\sum_p(a^\dagger_{0p}a_{0p}-\frac{1}{2}) \\ q_a\sum_p(-b^\dagger_{0p}b_{0p}+\frac{1}{2}) \end{array} \right\} + q_a\sum_{n=1}^{\infty}\sum_p(a^\dagger_{np}a_{np}-b^\dagger_{np}b_{np})\,, \quad \left\{ \begin{array}{c} (\eta_a\epsilon(B)>0) \\ (\eta_a\epsilon(B)<0) \end{array} \right\}. \tag{A.2}$$

The fermion sector of the thermodynamical potential (eq. (18)) is expressed in the classical level as

$$\Omega_0^{\rm fermi} = \frac{1}{2^{5/2}\pi^2}\zeta\left(\frac{3}{2}\right)\sum_a|q^aB|^{3/2} - \sum_a\frac{|q^aB|}{2\pi\beta}\left\{\sum_{n=1}^{\infty}\ln\left(1+e^{-\beta(\omega_n-q^a\mu^a)}\right) + \sum_{n=1}^{\infty}\ln\left(1+e^{-\beta(\omega_n+q^a\mu^a)}\right) + \ln\left(e^{\beta\mu_aq_a/2}+e^{-\beta\mu_aq_a/2}\right)\right\}, \tag{A.3}$$

where we assumed the fermion massless limit ($m_a = 0$). In the low temperature limit ($\beta\to\infty$), the second term of r.h.s. vanish and the result in ref. [7] is reproduced. In the case of $\beta^2|q^aB|\ll 1$, $\Omega_0^{\rm fermi}$ is calculated as [26]

$$\Omega_0^{\rm fermi} = \sum_a\left[-\frac{3}{4\pi\beta^3}\zeta(3)+\mathcal{O}\left(\beta|q_aB|^2\right) + \frac{q_a^2}{4\pi}\left\{\int_0^\infty dx\frac{1}{1+e^{\sqrt{x}}}+\mathcal{O}(|B|)\right\}\mu_a^2+\mathcal{O}(\mu_a^4)\right]. \tag{A.4}$$

We see that there is no linear B term in $\Omega_0^{\rm fermi}$ because of eq. (14).

The estimation of $\Delta\tilde{F}$ is approximately performed as follows. Since the $\tilde{\Pi}_{1Thermo}$ rapidly becomes small for large $|\vec{p}|$ by the factor $e^{-\vec{p}^2l_a^2}$, we can employ the cut-off l_{ave}^{-1} for the $|\vec{p}|$ integral in $\Delta\tilde{F}^{\rm boson}$, where $l_{ave}^{-2} =$

$\sum|\eta_a q_a^3\nu_a|/\sum|\eta_a q_a^2\nu_a|\cdot|B|$ is defined in ref. [8]. In the case of small l_{ave}^{-2}/κ^2 (namely small $|B|$), we evaluate $\Delta\tilde{F}^{\text{boson}}$ as

$$\begin{aligned}
\Delta\tilde{F}^{\text{boson}} &= \frac{1}{4\pi\beta}\int_0^{l_{ave}^{-2}} dy \ln\frac{1-e^{-\beta\sqrt{y+2N\omega_p^2}}}{1-e^{-\beta\sqrt{y+2N\omega_p^2+\kappa^2}}} \\
&= \frac{1}{4\pi\beta}\int_0^{l_{ave}^{-2}} dy\left\{\frac{1}{2}\ln\frac{y+2N\omega_p^2}{y+2N\omega_p^2+\kappa^2}+\mathcal{O}(y)\right\} \\
&= -\frac{1}{8\pi\beta}\frac{\kappa^2}{2N\omega_p^2}l_{ave}^{-2}+\mathcal{O}(l_{ave}^{-4}) \\
&= -\frac{\kappa e}{8\pi\ln 2}|B|+\mathcal{O}(|B|^2). \qquad (A.5)
\end{aligned}$$

Thus we can see in this rough estimation that the linear $|B|$ term with a negative coefficient appears.

APPENDIX 2:

The fermion propagator in the magnetic field, $S(x,y)=-i\langle 0|\mathrm{T}\psi(x)\bar{\psi}(y)|0\rangle$, satisfies the relation

$$S(x,y)_{qB<0}=U_C S(y,x)^t_{qB>0}U_C^\dagger. \qquad (A.6)$$

The Fourier transformation is given by

$$S_0(p)=\int\frac{d^3x}{(2\pi)^3}e^{ipx}S_0(x-y), \qquad (A.7)$$

where

$$S(x,y)_{qB>0}=e^{-i(x_1-y_1)(x_2+y_2)/2l^2}S_0(x-y)_{qB>0}. \qquad (A.8)$$

The relation of the propagator between $\eta_a=1$ fermions and $\eta_a=-1$ ones is

$$S_0(p)_{\eta_a=-1}=S_0(-p_0,\vec{p})_{\eta_a=+1}. \qquad (A.9)$$

The expression by the proper-time method is convenient in the calculation [7]. For the $\eta_a=1$-fermions, the expression for $\nu=0$ is

$$\begin{aligned}
S_0(p)^{\eta=+1}_{\nu=0} &= -il^2\int_0^\infty d\tau(\cos\tau)^{-1}e^{i(p_0^2-m^2+i\epsilon)l^2\tau-i\vec{p}^{\,2}l^2\tan\tau} \\
&\quad\times\left\{(m\cos\tau+ip_0\sin\tau)\,I+\left(-ip^1\sigma_1-ip^2\sigma_2\right)(\cos\tau)^{-1}\right. \\
&\qquad\left.+\,(p_0\cos\tau+im\sin\tau)\,\sigma_3\right\}. \qquad (A.10)
\end{aligned}$$

As to the case of $\nu = 1$, it takes the form

$$S_0(p)^{\eta=+1}_{\nu=1} = S_0(p)^{\eta=+1}_{\nu=0} + f(p), \tag{A.11}$$

where $f(p)$ is defined by $f(p) \equiv 2\pi i e^{-\vec{p}^{\,2}l^2}\delta(p_0 - m)(I + \sigma_3)$.

APPENDIX 3: Boson self-energies, Π_i's, in the weak field approximation

In our approximation, Π_i's are formally expressed as

$$\begin{aligned}
\Pi_0^{a\nu=0}(p,B) &= \frac{iq_a^2}{\vec{p}^2}\int\frac{d^3k}{(2\pi)^3} \\
&\times \frac{2\left[m_a^2 + k_0(k_0-p_0) + \vec{k}^2 - \vec{p}\cdot\vec{k}\right]}{(k^2 - m_a^2 + i\epsilon)\left[(k-p)^2 - m_a^2 + i\epsilon\right]} + O(B^2), &\text{(A.12)}\\
\Pi_1^{a\nu=0}(p,B) &= \frac{2i\eta_a q_a^2}{\vec{p}^2}\int\frac{d^3k}{(2\pi)^3} \\
&\times \frac{m_a\vec{p}^2 - i(p_0 - 2k_0)(k_1p_2 - k_2p_1)}{(k^2 - m_a^2 + i\epsilon)\left[(k-p)^2 - m_a^2 + i\epsilon\right]} + O(B^2), &\text{(A.13)}\\
\Pi_2^{a\nu=0}(p,B) &= \frac{4iq_a^2}{(\vec{p}^2)^2}\int\frac{d^3k}{(2\pi)^3} \\
&\times\frac{p_0^2\left[m_a^2 + k_0(k_0-p_0) + \vec{k}^2 - \vec{p}\cdot\vec{k}\right] + \vec{p}^2\left[m_a^2 - k_0(k_0-p_0)\right]}{(k^2 - m_a^2 + i\epsilon)\left[(k-p)^2 - m_a^2 + i\epsilon\right]} \\
&\qquad + O(B^2), &\text{(A.14)}\\
\Pi_0^{a\nu=1}(p,B) &= \Pi_0^{a\nu=0}(p,B) \\
&-\frac{4\pi q_a^2}{\vec{p}^2}\int\frac{d^3k}{(2\pi)^3}\frac{m_a + k_0}{k_0^2 - \vec{k}^{\,2} - m_a^2 + i\epsilon} \\
&\times\left\{e^{-(\vec{k}-\vec{p})^2l_a^2}\,\delta(k_0 - p_0 - m) + e^{-(\vec{k}+\vec{p})^2l_a^2}\,\delta(k_0 + p_0 - m)\right\}, &\text{(A.15)}\\
\Pi_1^{a\nu=1}(p,B) &= \Pi_1^{\nu=0}(p,B) \\
&+\frac{4\pi\eta_a q_a^2}{\vec{p}^2}\int\frac{d^3k}{(2\pi)^3} \\
&\times\left[\frac{(ip_1 - p_2)k_2 - (p_1 + ip_2)k_1}{k_0^2 - \vec{k}^2 - m_a^2 + i\epsilon}\,\delta(k_0 - p_0 - m_a)e^{-(\vec{k}-\vec{p})^2l_a^2}\right.
\end{aligned}$$

$$+\frac{-\vec{p}^2+(p_1-ip_2)k_1+(ip_1+p_2)k_2}{(k-p)^2-m_a^2+i\epsilon}\,\delta(k_0-m_a)e^{-\vec{k}^2 l_a^2}\Bigg], \quad (A.16)$$

$$\Pi_2^{a\nu=1}(p,B) = \Pi_2^{a\nu=0}(p,B)\,. \quad (A.17)$$

The straightforward calculation of (A.12) $\sim$ (A.17) reproduces the results in [5, 7] up to the order $\mathcal{O}(B_J^1)$;

$$\Pi_0^{\nu=0} = \frac{q_a^2}{8\pi}\frac{1}{(-p^2)^{1/2}}\left\{\sqrt{z}+(1-z)\sin^{-1}\frac{1}{\sqrt{1+z}}\right\}, \quad (A.18)$$

$$\Pi_1^{\nu=0} = -\frac{\eta_a q_a^2}{4\pi}\sqrt{z}\sin^{-1}\frac{1}{\sqrt{1+z}}, \quad (A.19)$$

$$\Pi_2^{\nu=0} = \frac{q_a^2}{8\pi}\frac{1}{(-p^2)^{1/2}}\left(\sqrt{z}+(1-z)\sin^{-1}\frac{1}{\sqrt{1+z}}\right), \quad (A.20)$$

where $z=-4m_a^2/p^2$ and

$$\Pi_0^{a\nu=1} = \Pi_0^{a\nu=0}-\frac{q_a^2}{2\pi}\frac{1}{p_0 l_a^2}\left\{\frac{1}{p^2+2m_a p_0}-\frac{1}{p^2-2m_a p_0}\right\}, \quad (A.21)$$

$$\Pi_1^{a\nu=1} = \Pi_1^{a\nu=0}-\frac{q_a^2}{2\pi}\frac{\eta_a}{l_a^2}\left\{\frac{1}{p^2+2m_a p_0}+\frac{1}{p^2-2m_a p_0}\right\}, \quad (A.22)$$

$$\Pi_2^{a\nu=1} = \Pi_2^{a\nu=0}. \quad (A.23)$$

APPENDIX 4: $\tilde{\Pi}_0$, $\tilde{\Pi}_1$ and $\tilde{\Pi}_2$ in weak field approximation

By using the replacement (21) and (22), the self-energies in the finite temperature system are formally obtained from eqs. (A.12) $\sim$ (A.17). After performing the angular integral included in $d\vec{k}$, the expression of $\tilde{\Pi}_i$'s takes the following form. For the $\nu=0$ case,

$$\tilde{\Pi}_0^{\nu=0}(p_3,\vec{p}^2,B,\beta) = -\frac{q^2}{\vec{p}^2}\frac{1}{2\pi\beta}\sum_{n=-\infty}^{\infty}\int_0^{\infty}dk\frac{k}{k_3^2+k^2+m^2}$$
$$\times\left\{1+\frac{k^2+m^2-p^2-(k_3-p_3)(3k_3-p_3)}{\sqrt{\{(k+|\vec{p}|)^2+m^2+(k_3-p_3)^2\}\{(k-|\vec{p}|)^2+m^2+(k_3-p_3)^2\}}}\right\}$$
$$+\mathcal{O}(B^2), \quad (A.24)$$

$$\tilde{\Pi}_1^{\nu=0}(p_3,\vec{p}^2,B,\beta) = \frac{-q^2\eta}{2\pi}\frac{m}{\beta}\sum_{n=-\infty}^{\infty}\int_0^1 dx \frac{1}{m^2+(k_3-p_3x)^2-x(1-x)\vec{p}^2}$$
$$+\mathcal{O}(B^2), \tag{A.25}$$
$$\tilde{\Pi}_2^{\nu=0}(p_3,\vec{p}^2,B,\beta)$$
$$= -\frac{p_3^2}{\vec{p}^2}\tilde{\Pi}_0^{\nu=0}(p_3,\vec{p}^2,B,\beta)_m + \frac{q^2}{\vec{p}^4}\frac{1}{2\pi\beta}\sum_{n=-\infty}^{\infty}\int_0^\infty dk \frac{k}{k_3^2+k^2}$$
$$\times\left[p_3^2 + \frac{p_3^2k^2-(4p^2+3p_3^2)k_3^2+(4k_3-p_3)(p^2+p_3^2)p_3-(4\vec{p}^2-p_3^2)m^2}{\sqrt{\{(k+|\vec{p}|)^2+m^2+(k_3-p_3)^2\}\{(k-|\vec{p}|)^2+m^2+(k_3-p_3)^2\}}}\right]$$
$$+\mathcal{O}(B^2). \tag{A.26}$$

As to the case of $\nu=1$,

$$\tilde{\Pi}_0^{\nu=1}(p_3,\vec{p}^2,B,\beta) = \tilde{\Pi}_0^{\nu=0}(p_3,\vec{p}^2,B,\beta) + \frac{q^2}{2\pi l^2 p_3}\left\{\frac{1}{p_3^2+\vec{p}^2-i2p_3m} - \frac{1}{p_3^2+\vec{p}^2+i2p_3m}\right\} + m\times\mathcal{O}(e^{-\vec{p}^2/(q_a|B|)}), \tag{A.27}$$

$$\tilde{\Pi}_1^{\nu=1}(p_3,\vec{p}^2,B,\beta) = \tilde{\Pi}_1^{\nu=0}(p_3,\vec{p}^2,B,\beta) + \frac{q^2\eta}{2\pi l^2}\left\{\frac{1}{p_3^2+\vec{p}^2-i2p_3m} + \frac{1}{p_3^2+\vec{p}^2+i2p_3m}\right\} + \mathcal{O}(e^{-\vec{p}^2/(q_a|B|)}), \tag{A.28}$$

$$\tilde{\Pi}_2^{\nu=1}(p_3,\vec{p}^2,B,\beta) = \tilde{\Pi}_2^{\nu=0}(p_3,\vec{p}^2,B,\beta). \tag{A.29}$$

APPENDIX 5: Calculation of $\tilde{\Pi}_{1\text{Thermo}}(p=0)$

The second term in r.h.s. in eq. (58) is given by

$$\tilde{\Pi}_{1\text{Thermo}}(p,B,\beta,\mu) = \frac{-1}{2\pi i}\frac{2\eta_a q_a^2}{\vec{p}^2}\int_{-\infty}^{\infty}d\lambda\int\frac{d^2\vec{k}}{(2\pi)^2}$$
$$\times\left\{\int_{-i\infty+\mu+\epsilon}^{i\infty+\mu+\epsilon}dk_0 f(k_0,\lambda)\frac{1}{e^{\beta(k_0-\mu)}+1}\right.$$
$$\left.+\int_{-i\infty+\mu-\epsilon}^{i\infty+\mu-\epsilon}dk_0 f(k_0,\lambda)\frac{1}{e^{-\beta(k_0-\mu)}+1} - \oint_C dk_0 f(k_0,\lambda)\right\}, \tag{A.30}$$

where

$$
\begin{aligned}
f(k_0,\lambda) \quad \equiv \quad & \frac{(ip_1-p_2)k_2-(p_1+ip_2)k_1}{k_0^2-\vec{k}^2-m^2+i\epsilon} e^{i(k_0-p_0-m)\lambda} e^{-(\vec{k}-\vec{p})^2 l_a^2} \\
& \frac{-\vec{p}^2+(p_1-ip_2)k_1+(ip_1+p_2)k_2}{(k_0-p_0)^2-(\vec{k}-\vec{p})^2-m^2+i\epsilon} e^{i(k_0-m)\lambda} e^{-\vec{k}^2 l_a^2}. \quad \text{(A.31)}
\end{aligned}
$$

The straightforward calculation produces

$$
\begin{aligned}
& \tilde{\Pi}_{1\text{Thermo}}(p,B,\beta,\mu) \\
& = +\frac{\eta_a q_a^2}{2\pi} \frac{p_1+ip_2}{\vec{p}^2} \sqrt{p_0(p_0+2m_a)} \{\theta(p_0)-\theta(-p_0-2m_a)\} \\
& \times \left\{ \frac{\theta(-p_0-m_a+\mu_a)}{1+e^{\beta(-p_0-m_a+\mu_a)}} - \frac{\theta(p_0+m_a-\mu_a)}{1+e^{\beta(p_0+m_a-\mu_a)}} + \theta(p_0+m_a-\mu_a) \right\} \\
& \times e^{-\vec{p}^2 l^2} e^{-p_0(p_0+2m_a)l^2} \int_0^{2\pi} d\phi e^{-i\phi} e^{2l^2\sqrt{p_0(p_0+2m_a)}(p_1\cos\phi+p_2\sin\phi)} \\
& + \frac{\eta_a q_a^2}{2\pi} \frac{p_1-ip_2}{\vec{p}^2} \sqrt{p_0(p_0-2m_a)} \{\theta(-p_0)-\theta(p_0-2m_a)\} \\
& \times \left\{ \frac{\theta(m_a-\mu_a)}{1+e^{\beta(m_a-\mu_a)}} - \frac{\theta(-m_a+\mu_a)}{1+e^{\beta(-m_a+\mu_a)}} - \theta(m_a-\mu_a) \right\} \\
& \times e^{-\vec{p}^2 l^2} e^{-p_0(p_0-2m_a)l^2} \int_0^{2\pi} d\phi e^{i\phi} e^{-2l^2\sqrt{p_0(p_0-2m_a)}(p_1\cos\phi+p_2\sin\phi)} \quad \text{(A.32)}
\end{aligned}
$$

To perform the integral for $d\phi$ in eq. (A.32), we expand the integrants by $|\vec{p}|$. Then the integrations become

$$
\begin{aligned}
& \int_0^{2\pi} d\phi e^{\mp i\phi} e^{\pm 2l^2\sqrt{p_0(p_0\pm 2m)}(p_1\cos\phi+p_2\sin\phi)}. \\
& \qquad = \pm 2\pi l^2 \sqrt{p_0(p_0\pm 2m)}(p_1\mp ip_2) + \mathcal{O}(\vec{p}^{\,2}). \qquad \text{(A.33)}
\end{aligned}
$$

By using eq. (A.33), we finally obtain eq. (60) from eq. (A.32).

APPENDIX 6

The formula to divide functions into the zero-temperature part and additional finite temperature part is [17]

$$\begin{aligned}\frac{1}{\beta}\sum_n f(k_0) &= \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} dk_0 f(k_0) + \frac{1}{2\pi i}\oint_C dk_0 f(k_0) \\ &-\frac{1}{2\pi i}\int_{-i\infty+\mu+\epsilon}^{i\infty+\mu+\epsilon} \frac{dk_0 f(k_0)}{e^{\beta(k_0-\mu)}+1} - \frac{1}{2\pi i}\int_{-i\infty+\mu-\epsilon}^{i\infty+\mu-\epsilon} \frac{dk_0 f(k_0)}{e^{-\beta(k_0-\mu)}+1},\end{aligned} \tag{A.34}$$

where $f(k_0)$ is a function without having singularities on imaginary axis and contour C is defined in Fig. 4.

Euler-Mclaurin's formula is given by

$$\begin{aligned}&\sum_{k=0}^{n} g(a+hk) = \frac{1}{h}\int_a^{a+nh} g(x)dx + \frac{1}{2}\{g(a)+g(a+nh)\} \\ &+\sum_{r=1}^{m-1}(-1)^{r-1}\frac{Br}{(2r)!}h^{2r-1}\left\{g^{(2r-1)}(a+nh) - g^{(2r-1)}(a)\right\} + \mathcal{O}(h^{2m}),\end{aligned} \tag{A.35}$$

where $B_r \equiv 2(2r)!/(2^{2r}-1)\pi^{-2r}\sum_{n=1}^{\infty}(2n+1)^{-2r}$ (the Bernoulli numbers).

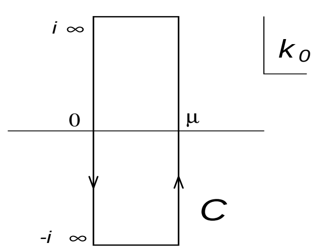

Figure 4: The contour C for the integral in eq. (A.34).

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